

# Mathematical modelling of basic anthropometric and mass inertial characteristics of young tennis players versus nonplayers: II case study for Bulgarian girls

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## Abstract

In this article, we present the findings from the anthropometric measurement of 55 non-players (GN) and 33 girls tennis players (GTP) ages 14 to 17. In order to create a mathematical 3D model of the average girl player (AGP) and nonplayer (AGNP), we determine each group's average mass and height. Every segment of the body is represented by a geometrical body in the model. Based on the model, we calculate each segment's mass-inertial characteristics, such as its volume, mass, centres of mass, and moments of inertia, using the analytical properties of these 3D geometric bodies. We make an effort to understand how the sport affects the physical development of teenage girl players, ages 14 to 17, by contrasting the results of the two groups. Our study is intended to inspire teenage girls to engage in sports, whether for fun or competition.

**Keywords:** Human body modelling, girls tennis players, mass-inertial parameters, anthropometry

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## 1. Introduction

In each country, the physical development of the population is used as an objective indicator of its health status and work capacity. Periodic monitoring of physical development in children and adolescents helps to establish regularities in biological and age-related changes, as well as the dynamics of their manifestation, under the influence of various exogenous or endogenous factors.

Its assessment is most often carried out by considering the leading anthropometric indicators of the body dimensions, shapes, and proportionality (height, body weight, head and body circumferences, diameters, lengths, skinfolds, etc.) during the different age periods. The metric characteristics of these indicators are used as a basis for developing age-normative anthropometric curves, which are helpful in medical practice [1, 2, 3]. Each sport is characterized by the specificity of the athlete's physical characteristics. Therefore, assessment of the anthropometric profile contributes to athletes' competitiveness and also is an important factor in raising the sports results [4, 5].

Khasawneh et al. [6] investigated the relationship between the anthropometric characteristics and body balance in static and dynamic positions of adolescent tennis players. The authors found that the width of the pelvis has the greatest importance for static balance, while for dynamic balance the lower leg circumference and the diameter in the ankle area are of leading importance in tennis players.

In sports medicine, complex studies of the athlete's physical development must necessarily include a method for evaluating and analyzing their anthropometric profile [7, 8]. In order to evaluate athletes' performance, enhance technique, and prevent injuries, coaching programs must now include measuring anthropometric and mass-inertial characteristics of the human body.

The purpose of this study is to develop a 16-segmental 3D mathematical model of the AGP and AGNP aged 14-17 years that can predict the mass-inertial properties of the human body. For the particular execution of the model in this investigation, we depend on data from our own anthropometric measurements. We compare the results of the two groups to clarify how sport affects the physical development of adolescent players.

## 2. Model and Method

A total of 88 adolescent girls (33 tennis players and 55 school girls, as a control group) aged 14-17 years, took part in the present study. All tennis players (TP) included in the study had trained in tennis for at least 2 years, for no less than 12 hours a week. The group of non-tennis players (NTP) included schoolgirls from some primary schools in Sofia, Bulgaria, who were not active in any sport. All girls and their parents volunteered for the research and gave their written informed consent. The study protocol was reviewed and approved by the Human Ethical Committee of the Institute of Experimental Morphology, Pathology and Anthropology with Museum – Bulgarian Academy of Sciences (Protocol № 3/11.04.2018) and conducted in accordance with the declaration of Helsinki for human studies of the World Medical Association [9].

The following anthropometric features were investigated by the use of Martin-Saller's classical method: body height, body weight, lengths, diameters, and circumferences of the body and limbs. Body lengths are defined as projections between the distances of specific anthropometric points according to the anthropometric methodology.

The 3D mathematical model of the human body used in the current study has been described in [10] – see there Fig. 1. We recall here that it consists of 16 segments (see, Table 1), following the anthropometric points used by [11], more precisely: head + neck, upper part of torso, middle part of torso, lower part of torso, thigh, shank, foot, upper arm, lower arm, and hand. The segments are modelled using relatively simple geometrical bodies. We assume complete "left-right" symmetry with respect to the sagittal plane.

We take the average values from our anthropometric investigation and design a model, which represents the so-defined AGP (height  $168.46 \pm 8.46$ , weight  $61.11 \pm 7.98$ ) and AGNP (height  $161.57 \pm 6.19$ , weight  $55.40 \pm 1.02$ ).

The geometrical models of segments and their defining parameters for both AGP and AGNP are shown in Table 1. The appropriate densities of the different segments [12] as well as those of the three parts of the torso [13] are also provided in the last column.

We advise the reader interested in more details to [14], where one already explained how to determine the numerical values of the geometrical parameters, select the anthropometric landmarks, modelling of the segments, etc. Abbreviations in Fig. 1 and Table 1 refer to the length of the corresponding segment  $L$ , the large radius of the corresponding geometric figures  $R$ , and the small radius  $r$  of those same geometric figures, respectively.

### 2.1. Volume and Mass of the Segments

We determine the volumes of the different segments using the formulas for the corresponding geometrical bodies.

For the head + neck modelled as an ellipsoid, the volume is calculated by the following formula:

$$(1) \quad V = \frac{4}{3} \pi r_{HE}^2 R_{HE}$$

The volume of the shank, foot, upper arm, and lower arm are approximated via the frustum of the cone:

$$(2) \quad V = \frac{1}{3} \pi l [R^2 + Rr + r^2]$$

where  $l$  is the length of the respective segment, and  $r$  and  $R$  are the semi-axes of its cross-section.

Due to the fact that the hand is assumed to be a sphere, its volume is

$$(3) \quad V = \frac{4}{3} \pi R_{HA}^3$$

The data for the volumes of the different segments for both groups being researched is presented in Table 2. After determining the volumes and mass densities of the segments extracted from the experiment, we proceed to compute the masses of these segments (see, Table 3).

Table 1

Geometrical parameters of the segments of the body, their approximations via geometrical bodies, as well as the densities of the different segments of 14-17-year-old girls tennis players and nonplayers

Body segments	Anthropometric parameters Girls Tennis Players [cm]	Anthropometric parameters Girls Nonplayers [cm]	Density [kg/m <sup>3</sup> ]
Head + Neck (Ellipsoid)	R <sub>HE</sub> = 15.6 r <sub>HE</sub> = 7.4	R <sub>HE</sub> = 14.6 r <sub>HE</sub> = 7.4	1087
Upper torso (Reverted right elliptical cone)	L <sub>TR</sub> = 41.3 L <sub>1</sub> = 13.8 R* = 17.7 r* = 10.4 R <sub>1</sub> = 13.1 r <sub>1</sub> = 7.7	L <sub>TR</sub> = 39.7 L <sub>1</sub> = 13.3 R* = 17.3 r* = 10.8 R <sub>1</sub> = 12.3 r <sub>1</sub> = 7.7	908
Middle torso (Elliptical cylinder)	L <sub>2</sub> = 14.4 R <sub>2</sub> = 13.1 r <sub>2</sub> = 7.7	L <sub>2</sub> = 13.8 R <sub>2</sub> = 12.3 r <sub>2</sub> = 7.7	1043
Lower torso (Elliptical cylinder + reverted elliptical cone)	L <sub>3</sub> = 6.6 L <sub>4</sub> = 8.7 R <sub>3</sub> = 13.1 R <sub>4</sub> = 13.1 r <sub>3</sub> = 7.7 r <sub>4</sub> = 7.7	L <sub>3</sub> = 6.4 L <sub>4</sub> = 8.4 R <sub>3</sub> = 12.3 R <sub>4</sub> = 12.3 r <sub>3</sub> = 7.7 r <sub>4</sub> = 7.7	1077
Upper arm (Frustum of cone)	L <sub>UA</sub> = 30.1 R <sub>SH</sub> = 4.0 R <sub>EL</sub> = 1.8	L <sub>UA</sub> = 29.8 R <sub>SH</sub> = 4.1 R <sub>EL</sub> = 1.8	1053
Lower arm (Frustum of cone)	L <sub>LA</sub> = 24.5 R <sub>EL</sub> = 4.0 R <sub>WR</sub> = 2.5	L <sub>LA</sub> = 23.1 R <sub>EL</sub> = 4.1 R <sub>WR</sub> = 2.4	1100
Hand (Sphere)	R <sub>HA</sub> = 4.1	R <sub>HA</sub> = 3.6	1137
Thigh (Frustum of cone)	L <sub>TH</sub> = 50.4 R <sub>TH</sub> = 8.8 R <sub>KN</sub> = 3.4	L <sub>TH</sub> = 48.6 R <sub>TH</sub> = 8.6 R <sub>KN</sub> = 3.4	1062
Shank (Frustum of cone)	L <sub>SK</sub> = 37.9 R <sub>KN</sub> = 3.4 R <sub>AN</sub> = 3.6	L <sub>SK</sub> = 36.8 R <sub>KN</sub> = 3.4 R <sub>AN</sub> = 3.5	1088
Foot (Frustum of cone)	L <sub>FO</sub> = 22.4 R <sub>FO</sub> = 3.5 R <sub>FE</sub> = 1.8	L <sub>FO</sub> = 21.9 R <sub>FO</sub> = 3.6 R <sub>FE</sub> = 1.8	1092
Body height	168.46±8.46	161.57±6.19	
Body mass	61.11±7.98	55.40±1.02	

\*Indexes: HE – head; TR – torso; TH – thigh; SK – shank; KN – knee; AN – ankle; FO – foot; FE – feet; UA – upper arm; LA – lower arm; SH – shoulder; EL – elbow; WR – wrist; HA – hand.

Table 2

Segment	Tennis players	Nonplayers
Head + Neck	3.6	3.3
Torso	13.7	12.6
Upper arm	1.4	1.5
Lower arm	0.8	0.7
Hand	0.3	0.2
Thigh	6.3	5.8
Shank	1.5	1.4
Foot	0.6	0.6

Volumes of the entire body and its segments, in [10<sup>-3</sup>m<sup>3</sup>], for girls tennis players and nonplayers.

Table 3  
The mass of the segments [kg] for girls tennis players and nonplayers.

Segment	Tennis Players	Nonplayers
Head + Neck	3.89	3.64
Torso	13.63	12.48
Upper arm	1.59	1.66
Lower arm	0.91	1.18
Hand	0.33	0.23
Thigh	6.66	6.20
Shank	1.58	1.50
Foot	0.63	0.62

## 2.2. Centre of Mass of the Segments

The majority of the expressions required for the mass centres can be derived in a relatively straightforward manner.

- For the upper torso, the centre of mass is given by

$$(4) \quad M_{CM}^{UT} = L_2 \frac{r_1(R_1 + R_2) + r_2(R_1 + 3R_2)}{2[r_1(2R_1 + R_2) + r_2(R_1 + 2R_2)]}$$

- For the middle torso, it is at

$$(5) \quad M_{CM}^{MT} = L_2/2$$

- For the lower torso, it reads

$$(6) \quad M_{CM}^{LT} = \frac{\frac{L_3}{2} + (L_3 + L_4/4) \frac{L_4}{3L_3}}{1 + \frac{L_4}{3L_3}}$$

Using equations (4)-(6), we can calculate the centres of mass for each segment of the body. Table 4 provides data on the relative locations of the centres of mass (the ratio between the distance from the proximal end of the segment and the length of the segment).

Table 4  
Relative location of the centre of mass of the body segments, i.e., the ratio between the distance from the proximal end of the segment and the length of the segment in percents [%], for girls tennis players and nonplayers.

Segment	Tennis players	Nonplayers
Head + Neck	50.0	50.0
Torso	43.6	42.7
Upper arm	50.0	50.0
Lower arm	42.5	47.8
Hand	50.0	50.0
Thigh	36.1	36.4
Shank	50.9	50.4
Foot	40.0	39.2

## 2.3. Inertial characteristics

As a final point, we calculated analytically and estimated numerically the segment's principal moments of inertia.

The following expressions are derived to obtain the principal moments of inertia for a frustum of an elliptical cone.

a) For  $I_{XX}^{CM}$

$$(7) \quad I_{XX}^{CM} = \frac{1}{240} \pi h \rho \left( R_2 (4h^2 + 3r_2^2)(3r_1 + 2R_1) + 3r_2(4h^2 + r_2^2)(4r_1 + R_1) - \frac{10(h r_2(3r_1+R_1)+h R_2(r_1+R_1))^2}{r_1(2r_2+R_2)+R_1(r_2+2R_2)} + 3r_2 R_2^2 (2r_1 + 3R_1) + 3R_2^3(r_1 + 4R_1) \right)$$

$$(8) \quad I_{YY}^{CM} = \frac{1}{240} \pi h \rho \left( 4h^2(3r_1(4r_2 + R_2) + R_1(3r_2 + 2R_2)) - \frac{10(h r_2(3r_1+R_1)+h R_2(r_1+R_1))^2}{r_1(2r_2+R_2)+R_1(r_2+2R_2)} + r_1^2 R_1(9r_2 + 6R_2) + 3r_1^3(4r_2 + R_2) + 3r_1 R_1^2(2r_2 + 3R_2) + 3 R_1^3(r_2 + 4R_2) \right)$$

and

$$(9) \quad I_{ZZ}^{CM} = \frac{1}{80} h \pi \rho \left( r_1^3 (4r_2 + R_2) + r_1^2 R_1(3r_2 + 2R_2) + r_1 (4r_2^3 + 3r_2^2 R_2 + 2r_2 R_2^2 + R_2^3 + R_1^2(2r_2 + 3R_2)) + R_1 (r_2^3 + 2r_2^2 R_2 + 3r_2 R_2^2 + 4R_2^3 + R_1^2(r_2 + 4R_2)) \right)$$

At this place  $R_1$  and  $R_2$  represent the semi-major principal axes of the base of the frustum of the cone,  $r_1$  and  $r_2$  indicate the semi-axes at the top of the frustum of the cone,  $h$  is its height and  $\rho$  is the experimentally determined density of the corresponding segment modelled as a frustum of cone.

Table 5 reports the results for the principal moments of the inertia of the AGP and AGNP obtained for all body segments.

Table 5  
Moments of inertia of the body segments through the centre of mass [kg.cm<sup>2</sup>] for girls tennis players and nonplayers

Segment	Tennis players			Nonplayers		
	I <sub>xx</sub>	I <sub>yy</sub>	I <sub>zz</sub>	I <sub>xx</sub>	I <sub>yy</sub>	I <sub>zz</sub>
Head + Neck	231.9	231.9	85.2	195.1	195.1	80.0
Upper torso	203.1	425.6	457.3.0	193.14	376.3	419.2
Middle torso	152.8	286.4	274.5	131.4	230.0	225.5
Lower torso	69.1	137.2	76.6	57.3	104.5	58.4
Lower arm	45.7	45.7	5.2	56.9	56.9	8.8
Hand	2.2	2.2	2.2	1.2	1.2	1.2
Thigh	1236.9	1236.9	162.9	1080.3	1080.3	145.3
Shank	194.7	194.7	9.7	173.4	173.4	8.9
Foot	34.1	34.1	2.5	27.6	27.6	2.7

Expressions (7) - (9) can be easily evaluated numerically and the principal moments of inertia for the three parts of the torso can be found using Steiner's theorem. Having in mind that a system of axes has been defined for every single segment, with an origin at the segment mass centre, whereas axes have been aligned with approximate body axes: frontal ( $x$ ), sagittal ( $y$ ), and longitudinal ( $z$ ).

Take notice that due to the  $x \leftrightarrow y$  symmetry in modelling the head, hand and upper and lower extremities, i.e. for all the segments excluding the torso and the thigh the principal moments of inertia  $I_{XX}$  and  $I_{YY}$  are equal to each other.

### 3. Discussion and Conclusions

In the current article a 16-segment 3D mathematical model of the human body, representative of the AGP and AGNP. The model is based on our own anthropometric data of 33 girls tennis players and 55 school girls (nonplayers).

We provide analytical expressions and numerically estimate the mass-inertial characteristics of all the body segments, including their volume, mass, centre of mass, and principal moments of inertia, after

deriving the necessary analytical expressions for the geometrical bodies used in the modelling.

In tennis, the role of inertial moments of the body is primarily related to generating and controlling rotational movements during various strokes and footwork. The moments of inertia of the upper body, particularly the trunk and shoulder region, are important in generating the necessary force and speed for an effective serve.

From the anthropometric measurements made, it can be seen that tennis players are 7 cm taller but six kilograms heavier than nonplayers. The high values of weight in tennis players girls is due to the high values of muscle mass accumulation in accordance with the sports training programs. From the obtained results, it can be seen that the moments of inertia of the nonplayer group are smaller than that of the tennis players, due to the different mass distribution in segments.

Regular participation in sports can help teenagers improve their cardiovascular health, muscle strength, and flexibility. Numerous health advantages may result from this, such as a decreased risk of diabetes, heart disease, and obesity. In addition, involvement in sports can help teenagers develop a sense of self-worth and confidence. This is because sports can provide opportunities for teenagers to succeed and achieve their goals. One other benefit is that sports can help teenagers maintain a healthy weight or lose weight if they are overweight or obese. The last seems of special importance for the self-esteem of girls which is, obviously, not the only gender-dependent difference the sport influences in girls with respect to boys.

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